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Detailed monitoring at Mount St. Helens since 1980 has enabled prediction of the intermittent eruptive activity (mostly dome growth) with unprecedented success. During 1981 and 1982, accelerating deformation of the crater floor around the vent (including radial cracks, thrust faults, and ground tilt) was the earliest indicator of impending activity. Numerical experiments using the finite element method to model the mechanics of the crater floor show that all deformation features can be explained by a uniform shear-stress boundary condition along the conduit wall. The magnitude of the shear stress required to match observed displacements (1–7 MPa) is inversely proportional to the conduit diameter (estimated to be 25–100 m). The most probable source of this shear stress is the flow of viscous magma up the conduit and into the lava dome. We propose a model in which the accelerating deformation, beginning as much as 4 weeks before extrusions, is caused by the increasing velocity of ascending magma in the conduit. This model is examined by using deformation data of the dome before four extrusions in 1981 and 1982 to estimate the volumetric flow rate through the conduit. This flow rate and an estimate of the effective viscosity of the magma enable calculation of an ascent velocity and an applied shear stress that, again, depend on the conduit diameter. The results of these calculations are consistent with the finite element experiments and show that the proposed model is feasible. In light of this model, events observed just before or near the time extrusions began, such as reversals of ground tilt direction from outward to inward and the sudden decrease in the number of shallow earthquakes, may indicate an abrupt decrease of shear stress in the conduit. This could be explained by a decrease in either the ascent velocity, or the effective viscosity of the magma ascending through the shallow conduit, or both, near the time of extrusion. Precursory deformation like that measured at Mount St. Helens should be observable at similar volcanoes elsewhere because it is caused by the fundamental process of magma ascent.

INTRODUCTION

Predictions of volcanic activity depend on the detection of reliable premonitory symptoms of forthcoming events. Since May 18, 1980, intermittent eruptive activity at Mount St. Helens has provided a rare opportunity to recognize precursors to eruptive events at a silicic volcano. All 13 eruptive episodes between June 1980 and December 1982 were predicted from tens of minutes to, more generally, a few hours beforehand primarily on the basis of seismicity [Malone et al., 1983]. Longer-term predictions were made 3 days to 3 weeks in advance of the last seven of these events (starting with that of April 1981) on the basis of deformation measurements [Chadwick et al., 1983; Dzurisin et al., 1983]. No false alarms were issued. This prediction record is "uncommon if not unparalleled in volcanology" [Swanson et al., 1983], and the ability to predict activity several weeks in advance is especially unique and noteworthy. At Mount St. Helens, the single most reliable means of predicting eruptive events this far in advance in 1981 and 1982 was monitoring the deformation of the crater floor.

While clearly useful for prediction, little is known about the cause of the deformation on which the predictions were based. One of the major goals of modern volcanology is to improve our knowledge of the causes of precursory phenomena, so that predictions become as deterministic as possible [Gorshkov, 1971; Decker, 1978; Swanson et al., 1983; Tilting and Bailey, 1985]. Therefore important questions that remain to be answered about the ground deformation at Mount St. Helens include (1) how can the deformation of the crater floor be explained mechanically?, (2) what can the deformation data tell us about the volcanic processes leading to extrusion?, and (3) are these precursory patterns unique to Mount St. Helens, or are they likely to be observed at other similar volcanoes? This study addresses these questions by constructing a simple mechanical model to explain the deformation of the crater floor, and then performing calculations to show that the model is credible and consistent with the data available.

This study is restricted to the years 1981 and 1982 when the crater floor was well exposed and monitored. Since 1982, the crater floor has been mostly covered by the spreading dome, rockfall debris, tephra, and snow, and precursory de-
formation has been measured primarily on the lava dome. Nevertheless, obvious deformation of the materials covering the floor suggests that it has continued to deform before and during eruptive events [Swanson, 1985].

**DEFORMATION OF THE CRATER FLOOR 1981-1982**

In 1981 and 1982, individual dome-building episodes were separated by 1–4 months of relative quiet. Periods of dome growth consisted of intrusion of magma into the dome for several weeks, culminated by the extrusion of lava onto its surface, lasting a few days. Surface deformation measured in the crater before extrusions included (1) the opening of radial cracks, (2) the movement along thrust faults, (3) radial displacement and tilting of the crater floor, and (4) swelling of the lava dome. All these movements accelerated before eruptive events.

**Cracks**

Ground cracks in the crater floor were first observed in September 1980 extending radiially outward from the dome (Figure 1a). Measurements of the distances across cracks showed that some cracks widened at rates that increased before subsequent activity [Swanson et al., 1981]. In addition, new cracks formed, existing cracks propagated away from the lava dome, and some cracks showed strike-slip components of movement (Figure 2a). The largest cracks grew to be 2–4 m wide and about 10 m deep [Chadwick et al., 1983].

**Thrust Faults**

During the December 1980 extrusion, thrust faults formed on the crater floor (Figure 1a). By the summer of 1981, a complex system of faults had disrupted much of the southwestern part of the floor (Figure 1b). The thrust faults were lobate and generally bounded by radial cracks that acted as tear faults. The thrusts initially formed as small buckles less than a centimeter high and typically moved a few meters before each subsequent extrusion, some eventually growing to have frontal scarps as high as 5 m that faced away from the dome. Parts of the floor bounded by thrust faults moved up and radially away from the dome (Figure 1c). The lateral movement of the thrusts was monitored by taping the distances between a point on the upper plate and two points on the lower plate. A network of survey points was leveled to monitor vertical displacements. Thrust movements accelerated systematically before extrusions (Figure 2b).

Fig. 1. (Opposite) Deformation of the crater floor at Mount St. Helens. (a) Sketch map of the crater floor made from air photos taken January 1981 showing radial cracks and thrust faults (teeth on upper plate). The lava dome is shown in gray, and the blocky pattern shows recent rockfall deposits from the crater walls. Lobes of lava that were emplaced during different dome-building extrusions are outlined. Rampart scarp is a fault scarp with relative movement up to the south that bounds the crater floor to the north. Note that radial cracks do not extend north of this fault. (b) Sketch map made from air photos taken September 1981. Dome talus is shown in coarse stipple. Vectors show displacements for rampart trilateration points (dashed) and thrust faults (solid) leading up to the extrusion on October 30, 1981. Note different vector scales. (c) Radial cross section from the center of the dome to the crater wall (x to x' in Figure 1b) showing displacements vectors across the upper plate of a thrust (relative to point z) before the extrusion on October 30, 1981. The vectors identified by the asterisk in Figures 1b and 1c are the same.
Fig. 2. Accelerating deformation on the crater floor and lava dome of Mount St. Helens leading to eruptive events. Plots show cumulative changes in distances measured (a) across a ground crack showing both dilational and strike-slip movements, (b) across the toe of a thrust fault, and (d) from a point on the crater floor to a point on the lava dome. (c) Tiltmeter data (radial component) show acceleration of outward tilting, turning to rapid subsidence 40 min before an explosion [after Dzurisin et al., 1983]. Vertical lines indicate the time of the start of eruptive activity (in Figures 2a and 2b the beginning of extrusion in September and October 1981; in Figures 2c and 2d, an explosion, followed by extrusion in March 1982). Monitoring stations are (a) Christina's radial, (b) Christina 2 thrust, (c) Roach tiltmeter, and (d) Hot Spot to Deloris.

Crater Floor Displacements and Tilting

Beginning in early 1981, points were periodically trilaterated from a baseline 1 km north of the dome in order to determine horizontal displacements near the vent. These measurements showed nearly radial movements (Figure 1b). Deformation of the floor and the dome began simultaneously, and horizontal displacements on the floor were cumulative and permanent; thus the conduit apparently remained filled with viscous magma between extrusions [Chadwick et al., 1983].

From May 1981 to August 1982, electronic tiltmeters on the crater floor monitored six extrusions. Outward tilting began several weeks before each extrusion, accelerated sharply for several days, and then abruptly changed direction to inward tilting minutes to hours before eruptive activity began (Figure 2c). Sites within 50 m of the dome (250–300 m from the vent) generally tilted a total of 2000–4000 (but as much as 20,000) microradians (μrad) before each extrusion [Dzurisin et al., 1983].

Dome Displacements

Frequent measurements of the distances and vertical angles between instrument sites on the crater floor and targets on the lava dome were initiated in October 1981 to detect expansion of the dome. Horizontal displacements as large as 30 m were measured, and demonstrated that the dome grew substantially by intrusion before lava was extruded onto its surface [Chadwick et al., 1983; Swanson et al., 1987]. The rate of dome expansion consistently accelerated before extrusions (Figure 2d).

Finite Element Experiments

Numerical experiments using theoretical mechanics and the finite element method have been performed to better understand the deformation of the crater floor. The finite
The crater floor is composed of tephra, pyroclastic fallback, and rockfall debris that are probably filling a conical vent excavated by the eruptive activity on May 18, 1980. The crater floor was separated from the floor of the breach by a fault scarp called "the rampart scarp" (Figures 1a and 3a). The scarp was first observed in June 1980 and became a prominent and persistent feature in the crater. It was vertical and trended approximately east-west with a 2 to 3-m relative displacement, up to the south. The significance of the rampart scarp is somewhat enigmatic, but it apparently was a manifestation of the boundary between the crater fill and the bedrock of the breach (Figure 3b). Evidence for this interpretation comes from aerial photographs taken days after May 18, 1980, which show that tephra on the crater floor was filling a circular depression, the north boundary of which is near the eventual location of the rampart scarp. Further evidence for a structural discontinuity is that radial cracks in the crater floor propagated from the conduit northward all the way to the rampart scarp, but never beyond (Figure 1a). By the end of 1982, most of the rampart scarp had been buried by the growing lava dome.

### Idealization for Numerical Modeling

**Geometry.** The conduit is assumed to be a vertical cylinder because displacements measured on the floor around the vent show a clear radial pattern (Figure 1b). The diameter of the conduit is constrained by field observations and previous theoretical work. The beginning of extrusion of a new lava dome (the oldest part of the present composite dome) was observed from a helicopter soon after the last major explosive activity in October 1980. The conduit that was feeding the dome was estimated to be about 25 m in diameter. Carey and Sigurdsson [1985] and Scandone and Malone [1985] calculated values of 95 m and 100-110 m, respectively, for the average diameter of the conduit from 7-km depth to the surface, based on modeling of the dynamics of the eruptive activity on May 18, 1980. The smaller value may be more relevant, since this study is only concerned with the upper 1 km of the conduit. In any case, these estimates give an upper and lower bound for the diameter of the conduit.

The crater walls are assumed to slope down beneath the crater floor at 55°, the same angle as that from the rim to the floor (Figure 3c). If the walls are modeled as sloping less than about 45°, surface displacements become increasingly concentrated near the vent and inconsistent with field data. The volume of the crater fill is therefore idealized as an inverted cone and the conduit a vertical cylinder at its center (Figure 4). For numerical modeling, we can take advantage of the vertical axis of symmetry and reduce this geometry to a two-dimensional, axisymmetric problem representing a vertical profile.

Two profiles with different geometry were used to distinguish between two areas of the crater fill: the part north of the vent that is bound by the breach and the rest of the fill that is bound by the crater walls. These will be re-
ferred to as the "breach" and "wall" profiles. The reasons for making this distinction are that (1) there was no thrust faulting north of the vent (Figure 1a) and the observed displacements there were an order of magnitude smaller than elsewhere (Figure 1b), (2) the breach is a relatively mobile boundary, whereas the walls are relatively fixed, and (3) the radial distance to the boundary is smaller to the north of the vent (Figure 1a). These distinctions are discussed further below.

The distance from the center of the conduit to the crater walls is about 500 m, and with walls sloping at 55°, the conduit wall adjacent to the crater fill is then slightly over 700 m (Figures 3c and 4b). However, the distance from the center of the conduit to the rampart scarp on the north side of the vent is only about 300 m. Using the same conduit length of 700 m, the north boundary of the crater fill slopes at 67° (Figures 3b and 4a).

**Boundary conditions.** In the wall profile, the crater walls are assigned a no-displacement boundary condition because deformation monitoring showed that the outer flanks of the volcano did not deform during 1981 and 1982 [Chadwick et al., 1983]. However, points that were monitored in the breach were displaced northward before some extrusions, suggesting that the boundary of the crater fill to the north is somewhat mobile. This condition is approximated in the breach profile by surrounding the crater fill with a stiff, but deformable, material to simulate the bedrock of the breach. This material is covered by a 100-m veneer with the same properties as the crater fill (Figure 4a).

Normal stress is added along the part of the ground surface beneath the dome to simulate its weight. We assume that the driving force for the deformation of the crater fill comes from within the conduit. The boundary condition along the conduit wall is therefore specified as either a normal stress, a shear stress, or both and can be varied along the length of the wall.

**Rheology.** The crater fill material is fragmental ejecta and rockfall debris which is unconsolidated at the surface and probably weakly consolidated at depth. This material is assumed to behave elastically, which means that strains are directly proportional to the applied stresses. A viscoelastic rheology (characterized by brittle behavior on short time scales and ductile behavior on long time scales) was considered, but not used for the following reasons: (1) the formation of the cracks and thrust faults shows that the crater fill deforms primarily in a brittle manner and not by viscous flow, (2) there is no evidence of relaxation phenomena that would suggest a viscoelastic response, and (3) for viscous effects to be measurable during the period between extrusions (a relaxation time of about 1 month), the crater floor would have to have an unreasonably low effective viscosity of $10^{15}$-$10^{16}$ Pa.

Two elastic constants are required to determine the relations between stress and strain in a linearly elastic, homogeneous, and isotropic material; Young's modulus and Poisson's ratio are used in this study. A value of $0.25$ is used for Poisson's ratio. The Young's modulus for the crater fill is not known. It is taken to be 100 MPa, which is within the range of values reported for soils [Vyalov, 1986] and slightly less than values for hyaloclastites, tuffs, and consolidated sandstones [Birch, 1966; Oddsson, 1981; Vyalov, 1986]. This value should be appropriate for the fragmental, unconsolidated to weakly consolidated tephra of the crater fill. In the breach profile, the Young's modulus of the breach bedrock is an order of magnitude higher than the modulus of the crater fill.

**Experimental Method**

The numerical experiments were conducted by inputing a variety of conduit radii and distributions of normal or shear stress applied on the conduit wall. Then the output was examined to see (1) if the stress field in the crater fill was compatible with the formation of radial cracks and thrust faults, and (2) how well the calculated displacements at the surface compared with the representative field data shown in Figure 1.

From the calculated stress field, stress trajectories can be plotted which are guides to orientations favorable to the formation of tensile cracks or zones of shear failure. Tensile cracks should form parallel to the axis of the maximum compressive stress. We assume that shear failure will occur on planes oriented 30° from the axis of the maximum stress.
compressive stress, as is predicted by Coulomb failure and is commonly observed in triaxial compression tests [Handin, 1966]. This technique of mapping stress trajectories to find the geometry of failure surfaces has been used in fault investigations elsewhere [Hafner, 1951; Hubbert, 1951; Sanford, 1959; Voight, 1976].

Thrust faulting was never observed on the north side of the dome. Thus the breach profile does not involve faulting, and model displacements can be compared directly with the trilateration data north of the dome in Figure 1b. However, faulting was common in the rest of the crater floor, and the wall profile must be altered to include thrust faults. This is done by lowering the Young's modulus between 1 and 2 orders of magnitude in individual elements along a narrow zone where shear failure is likely, as determined by the analysis of stress trajectories. Thrust faults are thus approximated in the numerical experiments by a zone of elements which are much weaker than the surrounding material. This method is qualitatively supported by the results of Mandl et al. [1977], who experimented with shear zones in granular material and found that they behaved as very weak zones across which most deformation took place, whereas the surrounding material remained essentially elastic and intact. Surface displacements calculated in this way can then be compared with displacements across the upper plate of the thrust fault in Figure 1c.

Finite Element Results

Stress fields. Either uniform normal or shear stresses applied to the conduit wall in both profiles create stresses in the crater floor that are appropriate for the formation of radial cracks. With an applied normal stress, the stresses in a radial direction are compressive throughout the floor, greatest adjacent to the conduit and decrease sharply with distance away from the conduit. With an applied shear stress the radial stress is tensile near the conduit and compressive elsewhere. In the wall profile, the stresses in the tangential direction (hoop stresses) begin as tensile stresses equal to or greater than magnitude to the radial stress at the conduit and become slightly compressive about three fourths of the way to the wall. In the breach profile, the tangential stresses remain tensile all the way to the rampart scarp.

These results suggest that cracks should initiate at the conduit wall and propagate outward in a radial direction all the way to the rampart scarp, to the north, but not all the way to the crater walls. These results are confirmed by field observations (Figure 1a). The results predict no shear stress along radial cracks; however, strike-slip movement was observed along many cracks in addition to dilational opening (Figure 2a). This strike-slip movement was a necessary accommodation of thrust faulting.

Radial stresses in the wall profile are higher, probably because of the fixed boundary, and this may be why thrusting occurs preferentially in areas of the crater floor where it is surrounded by the crater wall. The stress fields produced in the wall profile by either normal or shear-stress boundary conditions are appropriate for the formation of thrust faults. The geometry of the fault planes produced by the two boundary conditions is slightly different, however; normal stress creates a fault with a constant dip, whereas shear stress creates a fault that increases in dip with depth (Figure 5). Varying the conduit radius has little effect on the resultant stress fields.

Displacements. In both profiles, the magnitude of the shear stress required to match observed displacements is inversely proportional to the conduit radius. Thus a range of
stresses is possible in combination with the range of possible conduit radii.

Results from the wall profile show that displacements produced by the shear boundary condition using stresses of 1–7 MPa can match the field observations (Figures 6b and 7). However, the stresses required with the normal boundary condition, 11–120 MPa, are clearly unreasonably large. In both cases, the best fit to the field data is obtained with a fault zone 30 times weaker than the surrounding material. The model displacements are sensitive to the strength of the fault zone because if it is too weak, it deforms excessively and if it is not weak enough, there is little displacement across the zone. Nevertheless, the normal boundary condition consistently requires about an order of magnitude higher stresses than the shear condition to produce comparable displacements with a variety of fault zone strengths.

Results from the breach profile produce very similar results. The field displacements north of the dome can be matched satisfactorily by the shear boundary condition (Figures 6a and 7) using stresses of 1–7 MPa. In contrast, the normal boundary condition requires stresses of 5–40 MPa to produce large enough displacements, again probably unreasonably high. This is apparently because the shear boundary condition pushes the fill material toward the surface, where it is free to deform, whereas the normal boundary condition pushes it toward a barrier that restricts its mobility.

We conclude that the shear-stress boundary condition best reproduces the field displacements because the stresses required are reasonable and are very similar for the two profiles. Also the amount of ground tilting suggested by displacement vectors generated by the shear boundary condition from both profiles (1500–15,000 μrad) agrees well with values recorded by tiltmeters. These results do not preclude the existence of normal stress in the conduit but show that it would not contribute significantly to the surface deformation. In summary, the finite element results indicate that all the field observations, including the geometry of cracks and thrusts, as well as tilting of the crater floor and displacements measured on thrust faults, can be produced by a uniform shear stress between 1 and 7 MPa (10–70 bars) along the conduit walls.

A MECHANICAL MODEL FOR DEFORMATION OF THE CRATER FLOOR

The most likely physical source of shear stress along the walls of the conduit is the flow of magma up the conduit. Field evidence for shear in the magma during its migration to the surface includes (1) prominent planar flow banding with alternating layers of dense and vesicular dacite, (2) the alignment of long axes of xenoliths parallel to flow banding, and (3) disaggregation of xenoliths in planar shear zones and elongate cavities parallel to flow banding [Cashman and Taggart, 1983; Heliker, 1984]. The amount of shear stress applied on the conduit walls is related to the velocity of magma flow, so that an increase in the magma velocity would cause an increase in the displacement at the surface. In our model, increasing magma-ascent velocity is the fundamental cause of accelerating displacement rates at the surface.

TESTING THE MODEL

Can shear stress of the magnitude called for in the finite element experiments be produced by the magma rising in the conduit? We performed a series of calculations to test this model. Simplifications are necessary, so the aim of these calculations is only to determine whether the model is feasible and consistent with the data available.

During dome growth, magma is intruded into the dome before it is eventually extruded. The volume of magma that
rose in the conduit during a given time interval preceding an extrusion can be estimated by calculating the volume of intrusion into the lava dome during that time interval. Deformation measurements made on the dome can be used to calculate these volumes, \( V \), with the formula for an ellipsoidal segment

\[
V = \pi h \left( \frac{wl}{2} + \frac{h^2}{6} \right)
\]

in which \( h \) is the height, \( w \) is the half width, and \( l \) is the half length of the segment. An ellipsoidal segment is part of an ellipsoid cut off by a plane parallel to two of the semi-axes.

The volume of magma that has risen in the conduit, \( V \), divided by the time interval, \( t \), gives a volumetric flow rate, \( Q \). The time intervals are generally short enough that the flow rate is approximately constant during each interval. The average ascent velocity, \( \bar{v} \), is simply

\[
\bar{v} = \frac{Q}{\pi R^2}
\]

in which \( R \) is the radius of the cylindrical conduit. As a first approximation, the Mount St. Helens magma is assumed to be a Newtonian fluid, for which a linear relationship exists between the applied shear stress and the rate of strain. The maximum velocity, \( v_{\text{max}} \), is twice the average velocity for steady state laminar flow of a Newtonian fluid in a pipe [Bird et al., 1960].

The shear stress that would be applied to the conduit wall, \( \tau \), can be expressed using the viscosity of the magma, \( \mu \), and either \( v_{\text{max}} \) or \( Q \) [Bird et al., 1960]:

\[
\tau = \mu \frac{2v_{\text{max}}}{R} = \mu \frac{4Q}{\pi R^3}
\]

This shear stress can then be compared with the stress boundary condition necessary in the finite element calculations to match the displacements and tilt observed on the crater floor.

Magmas with crystals may behave more like a Bingham fluid with a yield strength than a Newtonian fluid [Shaw et al., 1968]. Murase et al. [1985] calculated an effective yield strength for the Mount St. Helens lava of 1.3 \( \times \) 10^5 Pa, and a similar value was calculated by Moore et al. [1978] for prehistoric lavas at Mount St. Helens with the same silica content. Using flow velocities near the time of extrusion calculated below, this yield strength would be exceeded even near the center of the conduit; the region of plug flow would be small and the velocity distribution near the wall would be the same as for a Newtonian fluid. Therefore a Bingham rheology with the above effective yield strength would have little effect on the stresses at the wall. It is also possible that the magma has a nonlinear rheology, but since very little is known about the flow parameters of Mount St. Helens dacite, a Newtonian rheology is used here.

**Physical Parameters and Assumptions in the Calculations**

The physical parameters on the right side of (3) (\( Q, \mu, \) and \( R \)) must be estimated to calculate the shear stress at the conduit wall. The radius of the conduit, as discussed earlier, is estimated to be 12.5–25 m based on field observations and theoretical calculations by other workers.

**Volumetric flow rate.** We used deformation measurements made on the dome leading up to four successive eruption episodes (October 1981, March 1982, May 1982, and August 1982) to calculate the volume of intrusion into the dome and the volumetric flow rates up the conduit. The data for the initial dimensions and volume of the dome for each event are taken from Table 1 of Swanson et al. [1987]. However, the volumes of the dome in this table do not include mantling talus, while the dimensions in the table do include talus. To find the best dimensions of an idealized ellipsoidal dome without talus but with the volume published by Swanson et al. [1987], we started with the dimensions including talus and decreased them systematically until they agreed with the talus-free volume. We incrementally decreased the half width and half length by twice as much as the height to take into account the fact that the talus is only on the sides of the dome but the topography on the dome is also irregular.

The idealized dome therefore begins with the volume published by Swanson et al. [1987]. After a time interval, a new volume is calculated by adding the measured dome displacements to the half width and half length. The difference of the two volumes is the volume of intrusion during that interval. This process is repeated for all measurement intervals. All the eruptive events except that of March 1982 had deformation measurements on four sides of the dome. The greatest displacements measured on the north and south sides were averaged and added to the half length, and those measured on the east and west sides were averaged and added to the half width. Interpolation was necessary to provide uniform time intervals if measurements were made on different sides on different days. A more accurate way of calculating the volume of intrusion added to the dome before extrusion would be to use topographic maps made from air photos. However, not enough photos and maps are available for this purpose.

The volume-supply rate to the dome apparently peaked at about the time that extrusion began. This is when the resolution in the data is poorest since measurements were rarely possible just before an extrusion, either because of poor weather or increased hazards. Deformation rates were increasing fastest then, so the last measurements before extrusions may give rates that are minimums. The available evidence suggests that the deformation stopped about the time extrusion began [Chadwick et al., 1983]. We therefore assume in these calculations that all of the measured deformation of the dome was completed by the onset of extrusion.

The time of the onset of extrusion was either observed or estimated from seismic and tilt data.

We calculated the volume-supply rate during extrusion by using a different, somewhat arbitrary method. The total volume of lava extruded was assumed to come up the conduit within 48 hours after extrusion began. The actual time that an extrusion ended was difficult to determine, because rates of extrusion declined in an exponential manner, but certainly most of the volume of each new flow came out in the first 48 hours. However, the volumetric flow rate in the extrusion interval was probably initially greater than the calculated rate because the calculated rate is averaged over 2 days.

**Effective viscosity of the magma.** During the dome-building eruptions in 1981 and 1982, lava eventually broke out near the summit of the dome and flowed down one of the sides for several days at decreasing rates. The most direct estimates of the effective viscosity of the lava are from
compressional ridges on the surface of flows and velocity gradients measured from the base to the top of active extrusions. Laboratory work by Murase et al. [1985] on a sample of dacite lava flow at Mount Trident, Alaska, yielded similar effective viscosity estimates between $10^{10}$ and $10^{11} \text{ Pa s}$. The maximum velocities for the Mount St. Helens flows were measured within the first 24 hours after extrusion began, but as the lava slowed over the next few days, the calculated effective viscosity increases to as much as $1 \times 10^{11} \text{ Pa s}$. However, this apparent increase may be due to the drag exerted by rapid cooling of the top and base of the flow. Another possibility is that the lava does not actually flow downslope under the influence of gravity, but rather is forced from behind in a pluglike manner and the measured velocities primarily reflect the rate of extrusion. Both of these processes may occur; however, arcuate compressional ridges on the surface of flows and velocity gradients measured from the base to the top of active extrusions provide clear evidence of flowage.

An effective viscosity of the magma between $10^{10}$ and $10^{11} \text{ Pa s}$, estimated from the direct field observations, appears to be reasonable when compared with empirical calculations. An empirical method developed by Shaw [1972] estimates the viscosity of a crystal-free liquid based on its composition and temperature. The matrix glass composition of the June 1981 lava as reported by Melson [1983] is used for this method and is assumed anhydrous. The temperature of the magma calculated by iron-titanium oxide geothermometry was $960 \pm 40^\circ \text{C}$ [Melson and Hopson, 1981; Melson, 1983]. This is consistent with the maximum recorded temperatures of fumaroles on the dome of $918^\circ \text{C}$. An estimate of the effect that crystals (40 vol% [Cashman and Taggart, 1983]) would contribute must be added to this crystal-free viscosity. One method, using an adaptation of Roscoe's equation [Marsh, 1981], gives an effective viscosity of $5.4 \times 10^9 \text{ Pa s}$ for the Mount St. Helens lava. Another empirical relation [Metzner, 1985], derived from experimental results from suspensions in polymeric liquids, yields an effective viscosity of $4.1 \times 10^{10} \text{ Pa s}$.

Laboratory work by Murase et al. [1985] on a sample of the Mount St. Helens dome gave an effective viscosity of $10^{15} \text{ Pa s}$ at a temperature of $1000^\circ \text{C}$ and $10^{11} \text{ Pa s}$ at $1100^\circ \text{C}$. However, measurements on a reheated sample may not truly represent the effective viscosity of a hot fluid lava that is rapidly cooling, crystallizing, and degassing.

**Results From Test Calculations**

The volumetric flow rates that were calculated for the four extrusions are similar with maximum rates before and during extrusion of $5-15 \text{ m}^3 \text{ s}^{-1}$ (Figure 8, Tables 2-5). The shear stress that would be applied on the conduit wall depends on the conduit radius and the effective viscosity of the magma (Figure 9, Tables 2-5). The primary conclusion to be drawn from the calculations is that peak shear stresses of 1–7 MPa along the conduit wall can be produced using the values of parameters discussed above. These stresses were shown earlier to be enough to produce the observed displacements on the crater floor in the finite element experiments.

The stresses required in the finite element results are inversely proportional to $R$; the stresses produced in the test calculations are inversely proportional to $R^3$ (see (3)). This means that only unique combinations of parameters can simultaneously satisfy both relationships. This is illustrated graphically in Figure 10, in which the curve intersections represent unique solutions. Other possible combinations of parameters are listed in Table 6.

Some differences exist between the four eruptive events and the completeness of the deformation measurements for each. In general, the thoroughness of the deformation measurements (in time and space) increases with each event. The last time interval before the October 1981 extrusion is long (5 days), and the measurements before the March 1982 event were from only one side of the dome; therefore the calculations for these events are less constrained than for the other events. In addition, the build up to the March 1982 extrusion was not well determined and the slope down which it was flowing may have been steeper than $30^\circ$.

**Table 1. Effective Viscosities Calculated From Lava Flows**

<table>
<thead>
<tr>
<th>Extrusion Date</th>
<th>Maximum Velocity, cm s$^{-1}$</th>
<th>Thickness, cm</th>
<th>Effective Viscosity, Pa s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct. 1981</td>
<td>0.125</td>
<td>3000</td>
<td>$4.2 \times 10^{10}$</td>
</tr>
<tr>
<td>March 1982</td>
<td>0.097</td>
<td>2500</td>
<td>$3.8 \times 10^{10}$</td>
</tr>
<tr>
<td>April 1982</td>
<td>0.111</td>
<td>2000</td>
<td>$2.1 \times 10^{10}$</td>
</tr>
<tr>
<td>May 1982</td>
<td>0.086</td>
<td>2500</td>
<td>$4.2 \times 10^{10}$</td>
</tr>
</tbody>
</table>
| Aug. 1982     | 0.493                       | 2000         | $4.8 \times 10^{10}$ *  

Data from Cascades Volcano Observatory (unpublished data, 1982).

The lava flows generally flowed down slopes of talus from the dome that were at or near the angle of repose. For this reason, $\alpha$ in (4) is assumed to be $30^\circ$ in all calculations.

* The effective viscosity calculated for this extrusion is probably less constrained than the others because the thickness of the flow was not well determined and the slope down which it was flowing may have been steeper than $30^\circ$. 

![Graph of volume-supply rate vs. time](image-url)  

**Fig. 8.** Log of volume-supply rate from Tables 2-5 plotted versus time leading up to four extrusions in 1981 and 1982.
TABLE 2. Calculations for the October 1981 Extrusion

<table>
<thead>
<tr>
<th>Measurement Interval</th>
<th>Dome Displacement, m</th>
<th>Volume Added to Dome, $x 10^6$ m$^3$</th>
<th>Q, m$^3$ s$^{-1}$</th>
<th>$V_{\text{max}}$, m h$^{-1}$</th>
<th>$\tau$, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>Time, LT</td>
<td>Length</td>
<td>Width</td>
<td>Incremental*</td>
<td>Cumulative</td>
</tr>
<tr>
<td>Oct. 22-23</td>
<td>1200-1200</td>
<td>0.096</td>
<td>0.190</td>
<td>0.019</td>
<td>0.019</td>
</tr>
<tr>
<td>Oct. 23-24</td>
<td>1200-1200</td>
<td>0.128</td>
<td>0.297</td>
<td>0.029</td>
<td>0.049</td>
</tr>
<tr>
<td>Oct. 24-25</td>
<td>1200-1200</td>
<td>0.154</td>
<td>0.332</td>
<td>0.033</td>
<td>0.082</td>
</tr>
<tr>
<td>Oct. 25-30§</td>
<td>1200-0800</td>
<td>2.935</td>
<td>10.000</td>
<td>0.907</td>
<td>0.989</td>
</tr>
<tr>
<td>Oct. 30–Nov. 1</td>
<td>0800–0800</td>
<td>0.000</td>
<td>0.000</td>
<td>2.000</td>
<td>2.989</td>
</tr>
</tbody>
</table>

Initial dome volume, 2.05 $x 10^7$ m$^3$. Initial dome dimensions (in meters): $l = 320.3$, $w = 260.3$, $h = 144.3$.

* For this extrusion, displacement measurements were possible only on the north side of the dome. It is assumed that the other three sides of the dome deformed identically.

† Values of $V_{\text{max}}$ shown for $R = 15$ m.

‡ Values of $\tau$ shown for $R = 15$ m and $\mu = 10^{10}$ P. See Figure 9 for $\tau$ values using other conduit radii.

§ Note the unusually long time interval; calculated values of $Q$, $V_{\text{max}}$, and $\tau$ are thus minimums.

The calculations above show that the flow of magma up the conduit is a feasible mechanism for producing sufficient...
TABLE 4. Calculations for the May 1982 Extrusion

<table>
<thead>
<tr>
<th>Measurement Interval</th>
<th>Dome Displacement, m</th>
<th>Volume Added to Dome, ( \times 10^6 \text{ m}^3 )</th>
<th>( Q, V_{\text{max}}, \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Date Time, LT</td>
<td>Length Width Incremental*</td>
<td>Cumulative</td>
</tr>
<tr>
<td>April 23–29</td>
<td>1200–1200</td>
<td>0.075 0.061</td>
<td>0.011</td>
</tr>
<tr>
<td>April 29–May 5</td>
<td>1200–1500</td>
<td>1.009 0.092</td>
<td>0.077</td>
</tr>
<tr>
<td>May 5–11</td>
<td>1500–1600</td>
<td>1.030 0.826</td>
<td>0.147</td>
</tr>
<tr>
<td>May 11–12</td>
<td>1600–1000</td>
<td>0.202 0.492</td>
<td>0.059</td>
</tr>
<tr>
<td>May 12–13</td>
<td>1500–1500</td>
<td>1.375 2.088</td>
<td>0.281</td>
</tr>
<tr>
<td>May 13–14</td>
<td>1500–0500</td>
<td>0.691† 1.902</td>
<td>0.225</td>
</tr>
<tr>
<td>May 14–16</td>
<td>0500–0500</td>
<td>0.000 0.000</td>
<td>2.500</td>
</tr>
</tbody>
</table>

Initial dome volume, \( 2.65 \times 10^7 \text{ m}^3 \). Initial dome dimensions (in meters): \( l = 352.1, w = 262.1, h = 166.1 \).
* This volume was added by intrusion during all measurement intervals except the last, when it was added by extrusion.
† Values of \( V_{\text{max}} \) shown for \( R = 15 \text{ m} \).
‡ Values of \( \tau \) shown for \( R = 15 \text{ m} \) and \( \mu = 10^{10} \text{ P} \). See Figure 9 for \( \tau \) values using other conduit radii.
§ This value is a minimum, since the displacement of the north side of the dome could not be measured during this interval or after extrusion.

shear stress to cause the precursory deformation of the crater floor before extrusions. However, it is difficult to say if any combination of physical parameters in Table 6 is preferable to another. Two pieces of evidence support a relatively small conduit radius. A velocity of 100–250 m h\(^{-1}\) for magma ascending in the conduit was derived independently on the basis of P wave travel time differences between two seismic stations preceding the May 1986 extrusion [Endo et al. 1987; E. T. Endo, personal communication, 1987]. This velocity is consistent with those calculated here using a radius of 10–15 m and an effective viscosity of \( 1 \times 10^{10} \text{ P} \) (Table 6). In addition, it is interesting to note that the total volume of intrusion into the dome is nearly the same for each event, about \( 1 \times 10^6 \text{ m}^3 \) (Tables 2–5), although considering the simplifications used to calculate them, these volumes are probably not constrained well enough to be confident of this observation. Nevertheless, this volume would be contained in a conduit with a radius of 12.5–15 m and a length of 2–3 km, the depth above which almost all precursory seismicity is concentrated [Malone et al., 1983; Endo et al., 1987].

TABLE 5. Calculations for the August 1982 Extrusion

<table>
<thead>
<tr>
<th>Measurement Interval</th>
<th>Dome Displacement, m</th>
<th>Volume Added to Dome, ( \times 10^6 \text{ m}^3 )</th>
<th>( Q, V_{\text{max}}, \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Date Time, LT</td>
<td>Length Width Incremental†</td>
<td>Cumulative</td>
</tr>
<tr>
<td>July 19–22</td>
<td>1200–1200</td>
<td>0.022 0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>July 22–29</td>
<td>1200–1200</td>
<td>0.054 0.085</td>
<td>0.012</td>
</tr>
<tr>
<td>July 29–31</td>
<td>1200–1200</td>
<td>0.011 0.017</td>
<td>0.002</td>
</tr>
<tr>
<td>July 31–Aug. 3</td>
<td>1200–1200</td>
<td>0.042 0.044</td>
<td>0.007</td>
</tr>
<tr>
<td>Aug. 3–6</td>
<td>1200–1200</td>
<td>0.042 0.029</td>
<td>0.006</td>
</tr>
<tr>
<td>Aug. 6–12</td>
<td>1200–1200</td>
<td>0.260 0.174</td>
<td>0.036</td>
</tr>
<tr>
<td>Aug. 12–16</td>
<td>1200–1600</td>
<td>0.230 1.033</td>
<td>0.116</td>
</tr>
<tr>
<td>Aug. 16–17</td>
<td>1600–1630</td>
<td>0.405 2.030</td>
<td>0.225</td>
</tr>
<tr>
<td>Aug. 17</td>
<td>1630–1845</td>
<td>0.120 0.449</td>
<td>0.052</td>
</tr>
<tr>
<td>Aug. 17–18</td>
<td>1845–0830</td>
<td>0.941 5.330</td>
<td>0.584</td>
</tr>
<tr>
<td>Aug. 18</td>
<td>0830–0900</td>
<td>0.057 0.112</td>
<td>0.015</td>
</tr>
<tr>
<td>Aug. 18–20</td>
<td>0900–1030</td>
<td>0.000 0.000</td>
<td>1.250*</td>
</tr>
</tbody>
</table>

Initial dome volume, \( 2.90 \times 10^7 \text{ m}^3 \). Initial dome dimensions (in meters): \( l = 366.1, w = 276.1, h = 167.1 \).
* For the calculation of the half-width dome displacements, data from the monitored point that moved the most on the west side (Near Miss 3) were not used, because measurements to nearby points showed that those data were not representative. Instead, data from a nearby target (Near Miss 1) are used.
† This volume was added by intrusion during all measurement intervals except the last, when it was added by extrusion.
‡ Values of \( V_{\text{max}} \) shown for \( R = 15 \text{ m} \).
§ Values of \( \tau \) shown for \( R = 15 \text{ m} \) and \( \mu = 10^{10} \text{ P} \). See Figure 9 for \( \tau \) values using other conduit radii.
¶ The last increment of intrusive volume is calculated from the last measurement interval (0830–0900) before extrusion began (at 1030). Some deformation of the dome occurred after extrusion began. This volume (0.25 \( \times 10^6 \text{ m}^3 \)) is added to the volume of extrusion (1.0 \( \times 10^6 \text{ m}^3 \)) for the final time interval, which starts at 0900 and ends 48 hours after extrusion began.
A. Shear: last interval before extrusion

Fig. 9. Maximum shear stress along the conduit walls (a) during the last measurement interval before extrusion and (b) averaged during extrusion, calculated as in Tables 2-5 ($\mu = 10^{10}$ P) but for a range of possible conduit radii. In Figure 9a the values for March 1982 and May 1982 are the same because they have the same extrusion volume.

B. Shear: averaged during extrusion

Fig. 10. Comparison of the stress required in the finite element experiments (Figure 7, breach profile) and the stress that can be produced by magma ascent according to the test calculations (Figure 9a, August 1982 extrusion; curve at left for $\mu = 10^{10}$ P, curve at right for $\mu = 10^{11}$ P). Arrows point to curve intersections which represent points that satisfy both relationships.

Implications for Magma Movement

The results presented thus far lead to the following interpretation of how magma moves toward the surface at Mount St. Helens. Chadwick et al. [1983] have suggested that the conduit remains filled with magma between eruptive episodes. The long-term supply rate of lava to the surface has been linear since the end of 1980, though the rate decreased by half at the end of 1981 [Swanson and Holcomb, 1985; Swanson et al., 1987]. This suggests that some constant rate process, possibly involving overpressurization, may be operating in the magma plumbing system at depth. The crystallization of groundmass and the subsequent concentration of volatiles in the remaining melt have been suggested as such a process [Cashman, 1987]. However, a "pressure valve" is apparently located between a reservoir of "eruptible" magma and a relatively narrow conduit to the surface. The distribution of shallow seismicity, noted above, suggests that this "valve" is 2-3 km deep (Figure 11a). The reservoir of eruptible magma may just be a wider portion of the conduit, since there is no evidence of a substantial magma body in the upper 7 km beneath Mount St. Helens [Scandone and Malone, 1985].

Once the "valve" is opened, a volume of magma proportional to the time since the last extrusion begins moving up the conduit [Swanson and Holcomb, 1985]; this begins a new precursory period. This volume of eruptible magma (relatively hot and gas rich) must force ahead of it the relatively degassed magma already in the conduit, and the degassed magma begins to intrude the dome (Figure 11b). Seismicity remains low during this time because the conduit stresses are not high enough to cause earthquakes [Malone et al., 1983]. As the magma ascends over the next 3-4 weeks, its velocity gradually increases. The removal of the degassed magma from above the ascending eruptible magma may contribute to this acceleration (Figure 11c). Eventually, the ascent velocity reaches a peak, and in a short time most of the volume of eruptible magma is delivered to the surface (Figure 11d).

Endo et al. [1987] concluded that magma moves up the conduit "in a matter of hours," but this is compatible with the scenario above because the ascent of magma is very slow and undetectable seismically during the first several weeks. Eventually, just before extrusion, the number of shallow earthquakes increases as the eruptible magma rises rapidly to the surface. The results of Frémont and Malone [1987] also support this interpretation. They concluded that clusters of nearly identical earthquakes ("multiplets") before or near the beginning of extrusions in 1984 and 1985 were caused by very high strain rates in a small volume around

<table>
<thead>
<tr>
<th>$\mu$, P</th>
<th>$R$, m</th>
<th>$\tau$, MPa</th>
<th>$V_{\text{max}}$, m h$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{10}$</td>
<td>10-15</td>
<td>5-8</td>
<td>100-150</td>
</tr>
<tr>
<td>$5 \times 10^{10}$</td>
<td>20-35</td>
<td>2-4</td>
<td>25-30</td>
</tr>
<tr>
<td>$1 \times 10^{11}$</td>
<td>30-50</td>
<td>1-3</td>
<td>10-15</td>
</tr>
</tbody>
</table>
the magma conduit. They also noted that the similarity of the events implied that the stress orientation remained constant during a multipier (12-72 hours) but that the increasing size of the events suggested that the stress was increasing, possibly due to "faster influx of magma." The first motions of these events were consistent with thrust faulting [Frémont and Malone, 1987].

At some point the factors governing the ascent of the magma cross a critical threshold, and the ascent velocity abruptly decreases. This takes place either just before or soon after extrusion begins. The volume-supply rate then decreases rapidly for a few days until extrusion stops and the "valve" closes again (Figure 11e). Individual eruptive events since 1983 have probably added larger volumes of intrusion to the dome than did those in 1981 and 1982. In these events, the intrusive volume may be a combination of the degassed magma that filled the conduit and eruptible magma that never was extruded.

**Implications for Tilt Reversals and Seismicity**

Several lines of evidence suggest that the stresses in the conduit decrease abruptly close to the time extrusion begins. First, ground tilt reversed direction from outward to inward minutes to hours before extrusions (Table 7 and Figure 2c). What caused these reversals is unclear, but the release of magmatic pressure has been a suggested hypothesis [Dzurisin el al., 1983]. Similarly, deformation measurements within hours of the onset of the extrusion of August 1982 showed that deformation of the crater floor had decelerated, whereas the dome was still rapidly deforming (Figure 12). The sudden decrease in the number of shallow earthquakes close to the time most extrusions began further suggests an abrupt decline in conduit stress [Malone et al., 1983]. This decrease in seismicity coincides with or occurs slightly after the reversal of tilt direction (Table 7).

A sudden drop in shear stress in the conduit just before extrusion begins could be explained by a decrease in either the velocity of magma ascent, or the effective viscosity of the magma rising through the shallow part of the conduit, or both. Which of these possibilities is more likely is unclear. The calculated ascent velocities during extrusion are greater than those before extrusion for the first three eruptive events (Tables 2-4). This is possible evidence favoring the viscosity hypothesis because the tilt reversals occur before extrusions. Such a decrease of effective viscosity could occur when the eruptible magma (hotter or more gas rich than the overlying degassed magma) finally is ascending through the upper
The precursory deformation at Mount St. Helens is caused by the fundamental process of magma rising toward the surface, so the results of this study should be applicable to other similar volcanoes. According to the proposed model, the critical parameters for the precursory deformation are the rate of volume supply, the diameter of the conduit near the surface, the effective viscosity of the magma, and the confined nature of the crater. This kind of deformation might be expected at other stratovolcanoes repeatedly erupting above a cylindrical vent in a crater filled with tephra.

A few structures that resemble the thrust faults at Mount St. Helens were observed at Soufriere Volcano, St. Vincent, along the perimeter of a lava dome after it was emplaced in 1979 [Sigurdsson, 1981; Shepherd et al., 1979]. These were interpreted to have been bulldozed up by the advancing dome (H. Sigurdsson, personal communication, 1985). Finite element experiments using shear stress beneath the dome as a boundary condition to simulate bulldozing show that this process cannot explain thrusting at Mount St. Helens because it cannot produce large enough vertical displacements. Large ground deformation and faulting have been documented for several eruptions in this century at Usu Volcano, Japan [Yokoyama et al., 1981]. The deformation during the 1977–1982 eruption included numerous strike-slip faults which accommodated thrusting and uplift amounting to over 150 m. This deformation has been modeled as being caused by the intrusion of a shallow cryptodome that never reached the surface [Katsui et al., 1985]. These examples support the notion that this kind of ground deformation is not unique to Mount St. Helens, and monitoring deformation around eruptive vents may be a valuable tool for prediction of eruptive activity at other silicic volcanoes.

CONCLUSIONS

Finite element experiments show that the deformation of the crater floor at Mount St. Helens precursory to eruptive activity in 1981 and 1982 can be explained by shear stresses of 1–7 MPa (10–70 bars) along the wall of a conduit between 25 and 100 m in diameter. The most likely source for this shear stress is from the flow of magma up the conduit and into the dome before extrusions. Calculations using defor-
mation data from the lava dome and physical parameters within their limits of uncertainty show that this mechanism is credible. The necessary shear stress could be produced by a magma with an effective viscosity of $10^{10}$ P flowing in a conduit with a diameter of 20–30 m, but if the effective viscosity is as high as $10^{11}$ P the diameter would have to be 60–100 m. The model suggests that magma begins its ascent about 4 weeks before extrusion, and its ascent rate must systematically increase to produce the accelerating deformation at the surface. Abrupt tilt reversals just before extrusions began, and a change in the character of seismicity near the time of extrusion, are interpreted to have resulted from a sudden drop of shear stress in the conduit. This could be explained by a decrease in either the ascent velocity or the effective viscosity of the magma ascending in the shallow conduit near the time of extrusion. Precursory deformation similar to that measured at Mount St. Helens should be observable at similar volcanoes.

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